Phase Space Analysis of Partial Differential Equations 2007

Certosa di Pontignano (Si)
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Abstracts

Serge Alinhac (Université de Paris 11, France)

Energy inequalities and general relativity

We discuss the issue of finding Morawetz type energy inequalities for wave equations corresponding to a given lorentzian metric; such inequalities imply local decay of the energy. This is done for perturbations of the Schwarzschild metric, including Kerr metrics. We emphasize the role played by the photosphere.

Luigi Ambrosio (Scuola Normale Superiore, Pisa, Italy)

Differentiability and rectifiability in Carnot groups

I will describe a recent joint work with B.Kleiner and E.Le Donne concerned with the rectifiability properties of sets of finite perimeter in general Carnot groups. In the talk I will also mention the relations between this problem and Rademacher-type theorems, and some recent applications to differentiability of maps with values in L^1.

Shiferaw Berhanu (Temple University, USA)

On analyticity of solutions of first-order nonlinear PDE

Let $(x, t) \in \mathbb{R}^m \times \mathbb{R}$ and $u \in C^2(\mathbb{R}^m \times \mathbb{R})$. We study the analyticity of solutions u of the nonlinear equation

$$u_t = f(x, t, u, u_x)$$

where $f(x, t, \zeta_0, \zeta)$ is complex-valued, real analytic in all its arguments and holomorphic in (ζ_0, ζ). We show that if the function u is a C^2 solution, $\sigma \in \operatorname{Char} L^u$ and $\frac{1}{i}\sigma([L^u, \overline{L^u}]) < 0$ or if u is a C^3 solution, $\sigma \in \operatorname{Char} L^u$, $\sigma([L^u, \overline{L^u}]) = 0$, and $\sigma([L^u, [L^u, \overline{L^u}]]) \neq 0$, then $\sigma \notin WF_a u$. Here $WF_a u$ denotes the analytic wave-front set of u and $\operatorname{Char} L^u$ is the characteristic set of the linearized operator

$$L^u = \frac{\partial}{\partial t} - \sum_{j=1}^{m} \frac{\partial f}{\partial \zeta_j}(x, t, u, u_x)\frac{\partial}{\partial x_j}.$$

Jean-Michel Bony (École Polytechnique, Paris, France)

Evolution equations and generalized Fourier integral operators

We consider evolution equations $\partial u/\partial t = ia^w(x, D)u$ where a is the (real valued) Weyl symbol of the operator $A = a^w$. For instance, Schrödinger-like equations. In this talk, after recalling what are generalized Fourier integral operators in the framework of the Weyl-Hörmander calculus, we give conditions on a and on the dynamics of its hamiltonian flow for having:

1. The operator a^w is essentially self-adjoint and the propagators e^{itA} are bounded between (conveniently related) generalized Sobolev spaces.
2. The propagators e^{itA} are generalized Fourier integral operators.

Jean-Yves Chemin (Université de Paris 6, France)

Hardy inequalities and applications

In this talk, we want to prove new Hardy inequalities in various situations. We give an application to the problem of restriction on an hypersurface of the Heisenberg group.

Paulo D. Cordaro (Universidade de São Paulo, Brazil)

Gevrey solvability in differential complexes associated to locally integrable structures

In this work we study some properties of the differential complex associated to a locally integrable (involutive) structure acting on forms with Gevrey coefficients. Among other results we prove that, for such complexes, Gevrey solvability follows from smooth solvability under the sole assumption of a regularity condition. As a consequence we obtain the proof of the Gevrey solvability for a first order linear PDE with real-analytic coefficients satisfying the Nirenberg- Treves condition $(\mathcal{P})$.

Nils Dencker (University of Lund, Sweden)

Solvability and subellipticity for systems

For scalar pseudodifferential operators of principal type, solvability is equivalent to condition (Ψ) by the resolution of the Nirenberg-Treves conjecture. This condition involves only the sign changes of the imaginary part of the principal symbol along the bicharacteristics of the real part. Subellipticity for an operator is equivalent to the Hormander bracket condition for the principal symbol.

For systems there are no corresponding results. In fact, condition (Ψ) for the eigenvalues of the principal symbol is neither necessary nor sufficient for solvability, except in the case of constant characteristics. Also, the conditions for subellipticity is not at all known except in the trivial diagonalizable case.

In this talk, we shall present some examples and results on solvability and subellipticity for systems of principal type. These are the systems whose principal symbol vanishes of first order on its kernel.

Isabelle Gallagher (Université Paris 7, France)

Some examples of large, global solutions to the Navier-Stokes equations

In this talk we will present some examples where large, global smooth solutions to the three dimensional Navier-Stokes equations may be constructed. We will start by recalling the most important and classical results on the Cauchy problem, and then we will show how the use of the special structure of the nonlinear term allows to go beyond those results. The most recent examples that will be presented are joint works with J.-Y. Chemin.

Patrick Gérard (Université de Paris 11, France

On the compactness defect of the Strichartz estimate for the wave equation

I shall describe the obstructions to the compactness of the L^4 Strichartz inequality for solutions to the wave equation in three space dimensions, in connection with the Lorentz invariance. Applications to the cubic wave equation will be discussed.

Mariano Giaquinta (Scuola Normale Superiore, Pisa, Italy)

On mappings into manifolds of bounded total variation

I shall discuss the notion of total variation for mappings into manifolds.

Nicholas Hanges (City University of New York, USA)

Hyperfunctions and (analytic) hypoellipticity

This talk is based on joint work with Paulo Cordaro. We discuss the problem of smooth and analytic regularity for hyperfunction solutions to linear partial differential equations with analytic coefficients. In particular we show that some well known "sum of squares" operators, which satisfy Hörmander's condition and consequently are hypoelliptic, admit hyperfunction solutions that are not smooth (in particular they are not distributions).

Jorge Hounie (Universidade de São Carlos, Brazil)

Boundary values of planar vector fields and Hardy spaces

It is classical that holomorphic functions $f(z)$ on the unit disc Δ with the property that the L^p norm, $0 < p \leq \infty$, of their restrictions to circles of radius $0 < r < 1$ is uniformly bounded, have boundary values ibf in the Hardy space $h^p(\partial\Delta)$. We discuss analogues of this behaviour for homogeneous solutions of locally integrable vector fields in the plane.

Enrico Jannelli (Università di Bari, Italy)

The hyperbolic symmetrizer - how to get rid of characteristic roots

The aim of this talk is to show that the hyperbolic symmetrizer is a useful tool to get well-posedness results for certain classes of linear hyperbolic equations, without directly assuming any hypothesis about characteristic roots.

Kunihiko Kajitani (Tokai University, Japan)

Time global solutions to the Cauchy problem for perturbed Kirchhoff equations

We consider the Cauchy problem for Kirchhoff equation corespondind to a elliptic and selfa-joint operator A which equals to Laplacian near the infinity and we find a unique solution in Sobolev spaces.

Herbert Koch (Universität Bonn, Germany)

Scattering for Kamdontsev Petviashvili II equation

The talk will give a survey on KPII and its relation to other dispersive equation. I will explain global well-posedness for small data in critical function spaces. Scattering will be an immediate consequence. The spaces of bounded p-variation of Wiener will play a key role in the analysis.

Gilles Lebeau (Université de Nice Sophia Antipolis, France)

Analysis of the Metropolis algorithm

The Metropolis algorithm is a basic tool of modern scientific computation. It was introduced by N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller and E. Teller in 1953 as a method for simulating non-overlapping hard discs in a bounded region. In this talk, we will present some results obtain in collaboration with P. Diaconis on the running time analysis for a local version of the Metropolis algorithm: the placement of a single ball of radius h randomly into a bounded subset of R^d equipped with a given probability density . We show that order of $1/h^2$ steps are necessary and sufficient for convergence from an arbitrary start. The proof uses spectral techniques and Nash inequalities.

Guy Métivier (Université de Bordeaux 1, France)

Instabilities in Zakharov equations

F.Linares, G.Ponce, J.-C.Saut have proved that a non-fully dispersive Zakharov system arising in the study of Laser-plasma interaction, is locally well posed in the whole space, for fields vanishing at infinity. We show that in the periodic case, seen as a model for fields non-vanishing at infinity, the system develops strong instabilities implying that the Cauchy problem is strongly ill-posed in Sobolev spaces. We get to same conclusion for the initial boundary value problem describing the space propagation of a laser beam entering in a plasma. An explicit amplification factor arises from the analysis, that can be computed for physical data.

Tatsuo Nishitani (University of Osaka, Japan)

An example of non effectively hyperbolic Cauchy problem not Gevrey well posed for any lower order term

In this talk we give an example of second order hyperbolic operator for which the Cauchy problem is not Gevrey 6 well posed for any lower order term. This phenomina is caused by the existence of a null bicharacteristic landing on the double characteristic manifold.

Vesselin Petkov (Université de Bordeaux 1, France)

Singularities of the scattering kernel related to trapping rays

An obstacle $K \subset \mathbb{R}^n$, $n \geq 3$, n odd, is called trapping if there exists at least one generalized bicharacteristic $\gamma(t)$ of the wave equation staying in a neighborhood of K for all $t \geq 0$. We examine the singularities of the scattering kernel $s(t, \theta, \omega)$ defined as the Fourier transform of the scattering amplitude $a(\lambda, \theta, \omega)$ related to the Dirichlet problem for the wave equation in $\Omega = \mathbb{R}^n \setminus K$. We prove that if K is trapping and $\gamma(t)$ is non-degenerate, then there exist reflecting (ω_m, θ_m)-rays δ_m, $m \in \mathbb{N}$, with sojourn times $T_m \to +\infty$ as $m \to \infty$, so that $-T_m \in \operatorname{sing\,supp} s(t, \theta_m, \omega_m)$, $\forall m \in \mathbb{N}$. We apply this property to study the behavior in $\mathbb{C}$ of cut-off resolvent and the scattering amplitude for trapping obstacles.

Michael Reissig (TU Bergakademie Freiberg, Germany)

The precise loss of regularity for weakly hyperbolic Cauchy problems

We consider the family of weakly hyperbolic Cauchy problems

$$u_{tt} - \lambda^2(t) b^2(t) u_{xx} - a \frac{\lambda^2(t)}{\Lambda(t)} u_x = 0, \; a \in \mathbb{R}, \, \Lambda(t) = \int_0^t \lambda(s) ds,$$
$$u(0, x) = u_0(x), \; u_t(0, x) = u_1(x).$$

The coefficient in front of u_{xx} consists of two parts: The function λ describes the only degeneracy in $t = 0$, the function b describes the oscillating behaviour near $t = 0$. The sharp Levi condition is satisfied with respect to the degeneracy. In general one expects the so-called *loss of regularity* of solutions in comparison with the regularity of data. But we want to improve some known examples to find out under which conditions we have no loss of regularity. Thus we need to find an approach which gives the precise loss. We discuss moreover the optimality of our result and thus the question if the finite loss really appears.
Joint work with Lu Xiaojun (Zhejiang University Hangzhou).

Fulvio Ricci (Scuola Normale Superiore, Pisa, Italy)

Joint spectral multipliers in the Schwartz class for commuting differential operators on the Heisenberg group

We present the following recent result, obtained jointly with F. Astengo and B. Di Blasio:

let L be the sublaplacian and T the central derivative on the Heisenberg group H_n, and let $m(\lambda, \xi)$ be a Borel function on the joint L^2-spectrum of L and $i^{-1}T$. Then $m(L, i^{-1}T)f = f * K$ for a kernel K in the Schwartz class if and only if m extends to a Schwartz function on $\mathbb{R}^2$.

We also illustrate some generalizations in terms of Fourier-Gelfand transforms for Gelfand pairs.

Sergio Spagnolo (Università di Pisa, Italy)

Hyperbolic systems with symmetrizers of diagonal type

We shall discuss some recent results, obtained in collaboration with G. Taglialatela, and with T. Kinoshita, which are concerning the existence of a diagonal or "nearly diagonal" symmetrizer for some classes of hyperbolic systems.

Giorgio Talenti (Università di Firenze, Italy)

On complex-valued $2D$ eikonals

Geometrical optics fits well a variety of issues, but especially provides an archetypal asymptotic theory of monochromatic high-frequency electromagnetic fields. More powerful theories of such fields have been worked out by Felsen, Kravtsov, Ludwig and their followers. One is enough for modeling basic optical processes, such as the propagation of light, reflection and the development of caustics. The others both embrace geometrical optics and are additionally apt to account for certain optical phenomena, such as the rise of evanescent waves past a caustic, that are beyond the reach of geometrical optics. A leitmotif of these is letting an appropriate parameter, which plays the role of an eikonal to all intents and purposes and rules a certain asymptotic expansion of the electromagnetic field, encode information on both phase and amplitude – in fact, take complex values. The following mathematical principle is ultimately behind the scenes: any geometric optical eikonal, which is generated via the conventional mechanism of geometric optical rays and eventually fights some caustic, can automatically be continued past the caustic provided both a real part and an extra non-zero imaginary part be called into play.

The following partial differential equation

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = n^2(x,y) \tag{1.1}$$

underlies the mentioned theories. Here x and y denote *rectangular coordinates* in two-dimensional Euclidean space; n is a *real-valued, strictly positive* function of x and y; w is allowed to take both *real* and *complex* values. Function n represents the *refractive index* of an appropriate two-dimensional medium – its reciprocal stands for velocity of propagation. Function w is named *eikonal* according to usage, and relates to the asymptotic behavior of an electromagnetic field as the wave number grows large – the real part of w accounts for oscillations, the imaginary part of w accounts for damping.

Let u and v be real-valued functions of x and y. The following partial differential system

$$\begin{aligned} &u_x^2 + u_y^2 - v_x^2 - v_y^2 + n^2(x,y) = 0, \\ &u_x v_x + u_y v_y = 0 \end{aligned} \tag{1.2}$$

characterizes those complex-valued solutions to (1.1) that obey

$$w = v + iu. \tag{1.3}$$

Observe the architecture of (1.2): gradients are involved through their orthogonal invariants only. Observe also the following properties, which result from a standard test and easy algebraic manipulations. First, system (1.2) is *elliptic-parabolic* or *degenerate elliptic*. Second, a solution (u,v) to (1.2) is *elliptic* if and only if its former component u is free from critical points. In the present paper we assume the refractive index is conveniently smooth, and review some results that have been recently obtained on either complex-valued solution to equation (1.1) or sufficiently smooth solutions to system (1.2).

David S. Tartakoff (University of Chicago, USA)

Gevrey hypoellipticity for non-subelliptic operators

After Kohn's celebrated example of a hypoelliptic, but not subelliptic operator which loses many derivatives, and Christ's example of adding one more variable and killing hypoellipticity altogether, it is natural to ask for Oleinik-type operators with powers of x or $|z|$ before the added term. We obtain Gevrey hypoellipticity results in this setting.

François Treves (Rutgers University, USA)

A class of multidimensional integrodifferential equations with infinitely many constants of motion

The lecture will present preliminary results on applications of the theory of noncommutative KdV equation $u_t = \partial^3 u - 3\left(u\partial u + (\partial u)\, u\right)$ (recently developed by the author) to algebras of matrices, first of finite rank and then of infinite rank. The resulting differential equations in these algebras can only make sense in a noncommutative set-up, as the basic "space derivation" is commutation with another (fixed) matrix. The infinite rank situation is reinterpreted, via Hermite expansion, in the algebra of bounded linear operators on Schwartz space $\mathcal{S}(\mathbb{R}^n)$. Special choices of the "space derivation" as commutation with partial differential operators can be identified to evolution equations whose linear part is partial differential (in $\mathbb{R}^{2n+1}$) and the nonlinear part is integrodifferential: a partial differential operator (in $\mathbb{R}^{2n}$) acting on the square of the unknown u in the sense of Volterra composition. The choice of the harmonic oscillator $D_x^2 + x^2$ (when $n = 1$) is particularly amenable to Hermite expansion approach. Existence and uniqueness of global solutions in the Cauchy problem can be proved for special initial data (in $\mathbb{R}^{2+1}$).

Claude Zuily (Paris 11, France)

Smoothing effect for Schrödinger equation with unbounded potentials on exterior domains

In this talk (a common work with L.Robbiano) I will discuss the Kato smoothing effect for solutions of the Dirichlet problem associated with a Laplacian on a Riemanian manifold perturbated by an at most quadratic potential.